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Assessing the Welfare Cost of a Fixed Exchange-Rate Policy*

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Abstract

This paper performs a welfare analysis based on the hypothetical scenario that Denmark gave up its peg and started conducting monetary policy according to a Taylor rule. For this we rely on a dynamic stochastic general equilibrium model for a small open economy that was estimated on Danish data using Bayesian methods. We obtain the result that the gain in welfare is equivalent to a permanent increase of around 0.8 pct in the level of consumption. Examining a range of alternative scenarios does not change this conclusion, unless we assume a degree of policy errors under the Taylor rule that is substantially larger than those estimated by other studies.

Keywords: Open economy, Monetary policy, Business cycles, Welfare

JEL Classifications: E3, E4, E5, F4

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1 Introduction

With the recent enlargement of the European Union there is now a sizeable number of countries bordering the euro area who are facing a complex question on their future monetary policy. In the longer term, the question will be whether these countries should adopt the euro or conduct an independent monetary policy as Sweden and Great Britain have been doing with considerable success. Recent papers on the optimality of currency areas versus independent monetary policies include Benigno and Benigno (2000) and Benigno (2004).

Yet, it remains an open question how long the new EU members would have to wait before they could fully join the monetary union. Arguably, they could be in a waiting position for years where they will be pegging the euro and thus essentially be passively adopting the monetary policy conducted by the ECB. Thus, it would be of general interest to seek to quantify the welfare consequences of pegging the euro compared with an independent monetary policy regime. Due to the combination of dramatic changes in their economies over the last decade and a very limited set of time series on key aggregate measures, obtaining reliable estimates on the welfare implications of different monetary policy regimes for the new EU members from central and eastern Europe is, alas, a very difficult task.

Incidentally, Denmark offers an interesting case study on this exact question. Although a member of the ERM for years, Denmark has opted out of the third stage of the EMU for political reasons (which mainly has to do with an EU-skeptic population). As a consequence, Denmark has effectively had a fixed exchange-rate policy for decades now; thus, since 1987 the monetary policy has kept a constant parity on the D-mark/euro. This paper seeks to quantify the welfare implications of this peg regime compared with a hypothetical independent monetary policy regime which seeks to stabilise inflation and output volatility. Thus, since the Danes have twice rejected to adopt the euro, this paper provides an answer to the question of which alternative monetary policy is the optimal one.

In order to address this question, we formulate a dynamic stochastic general-equilibrium (DSGE) model for the Danish economy and calculate a second-order approximation around its steady state. We have chosen this solution method since first-order approximations are not adequate for welfare analysis of stochastic models, cf. Kim and Kim (2003) and Schmitt-Grohe and Uribe (2004b).

The model itself builds on the one presented in Kollmann (2002). However, while Kollmann bases his welfare analysis on a calibration of the structural parameters of his model, we rely on the model that was estimated on Danish data in Dam and Linaa (2005). This model makes three important departures from the one in Kollmann (2002). Firstly, in the fixed-exchange rate case we do not consider a peg that is perfect; instead we postulate that the central bank is only able to keep the exchange rate stable up to an exogenous shock, reflecting the (minor) fluctuations observed in the exchange rate around its parity. Secondly, we replace Kollmann's assumption of a competitive labour market with one of differentiated labour and monopolistic competition amongst the households leading them to raise wages above the competitive level; in addition, we impose wage rigidities à la Calvo (1983) by assuming that households are unable to revise their wage

demands every period. Thirdly, we generalise the utility function applied in Kollmann (2002) so that the key elasticities as well as habits reflecting household preferences are estimated. All in all, the model underlying our analysis has richer dynamics which *ceteris paribus* improves its empirical plausibility. This should facilitate the reliability of the quantitative welfare cost that we deduce in this paper.

There are potentially important matters not included in the current analysis; if Denmark decided to adopt a Taylor rule, risk aversion from foreign investors might induce a reduction in direct investment flows into Denmark caused by an increased uncertainty regarding the exchange rate. Furthermore, also Danish exporters face uncertainty regarding the exchange rate and could need to engage in costly arrangements with financial intermediaries in order to eliminate this uncertainty when trading with agents abroad. Finally, we ignore issues related to the potential budget discipline being put on the Government in order to keep a peg credible.

Abstaining from these issues we conclude that there are benefits to be attained from letting monetary policy be conducted according to a Taylor rule (cf. Taylor, 1993; Woodford, 2003) instead of maintaining the peg which is the current goal of Danish monetary policy. Our estimate suggests that the gain in welfare is equivalent to a permanent increase of 0.8 pct in the level of consumption. The optimal Taylor rule is found to be characterised by attaching a weight of 3 (which is the ceiling of our grid search) to inflation and a weight of 0.8 on output growth. Contrary to Schmitt-Grohe and Uribe (2004b) we do not find it beneficial for the central bank to smooth interest rates over time.

With regards to the causes of the higher level of welfare under the Taylor rule, we obtain mixed results: in terms of consumption, the higher welfare is founded in the higher mean of consumption under the Taylor rule, although the volatility of consumption has also increased. For labour this result is reverted; under the peg regime labour is more volatile than under the Taylor rule, while the mean is predicted to be lower under the peg. Overall, agents prefer the higher consumption, despite higher volatility and more labour efforts.

Two related studies are Ambler et al. (2003) and Schmitt-Grohe and Uribe (2004a). Ambler et al. (2003) apply maximum-likelihood techniques to estimate a DSGE model without capital of a small open economy and search for the optimal Taylor rule. They do not, however, consider a fixed exchange-rate regime, as the benchmark model in their study is a Taylor rule estimated on Canadian data. They obtain the result that the gain in welfare is equivalent to a permanent increase of 1.4 pct in the level of consumption compared with the level of welfare under the historical Taylor rule. Schmitt-Grohe and Uribe (2004a) analyse the closed-economy model laid out and estimated by Christiano et al. (2001). Contrary to existing studies, Schmitt-Grohe and Uribe (2004a) find that inflation should be attached a value of just 1, giving room for what they style “a significant degree of optimal inflation volatility”. This is explained by the presence of indexation to past inflation.

This paper goes on as follows: In Section 2 the model is laid out, and in Section 3 it is parameterised. In Section 4 the welfare measure and the solution method is being described and

in Section 5 we use this to find the optimal Taylor rule. In Section 6 the results are presented and in Section 7 we analyse the robustness of the results. Section 8 concludes.

2 Model

The model is basically identical to the one used by Dam and Linaa (2005) which again draws heavily on the model presented in Kollmann (2002). Like him, we consider a small open economy that produces a continuum of intermediate goods which are aggregated and sold under imperfect competition to final-good producers at home and abroad. Producers of intermediaries only reoptimise prices infrequently a la Calvo (1983), but can differentiate fully between the domestic and foreign market and price their goods abroad in the local currency. It follows that prices are sticky in the currency of the buyer, an assumption that has been forcefully argued by, e.g., Betts and Devereux (1996, 2000). Recently, Bergin (2003, 2004) has compared local and producer currency pricing in estimated DSGE models and found strong empirical support for local currency pricing. Final goods are produced from aggregates of the intermediate goods from home and abroad and sold in a perfectly competitive market. Thus, all trade takes place in intermediary goods.

We replace the homogenous and perfectly competitive labour market of Kollmann (2002) with one of differentiated labour services and rigid wage setting due to Erceg et al. (2000) and Kollmann (2001) which was also implemented in the Christiano et al. (2001) model (henceforth the CEE model). Furthermore, we follow Smets and Wouters (2003) and assume CRRA preferences and external habit formation; thus, the preferences analysed in Kollmann's model are a special case of ours. We maintain, however, the quadratic investment adjustment costs in the relative level of capital, the debt premium on the interest earned on foreign bonds and the UIP shock from the Kollmann (2002) model. Finally, we introduce an imperfect peg regime for monetary policy with a persistent policy shock.

An important deviation from Dam and Linaa (2005) is that in this paper we treat mark-up rates as constants rather than allowing them to follow a stochastic process. The reason is a technicality; our method of obtaining a second-order approximation requires that we write the non-linear system as a multivariate first-order expectational difference equation. To our knowledge it is not possible to write a model with stochastically varying markups in this form, and thus we introduce constant markups. Schmitt-Grohe and Uribe (2004a) made an equivalent simplification when they considered optimal monetary policy in the CEE model.

In this section we outline the various components of the model.¹

2.1 Households

Like Erceg et al. (2000) we assume a continuum with unity mass of symmetric households who obtain utility from consumption of the final good and disutility from labour efforts. Thus, they are

¹A technical appendix with a thorough derivation of the model is available upon request.

all characterized by the following preferences:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right], \quad (1)$$

$$U(C_t^*, l_t(j)) = \zeta_t^b \left[\frac{C_t^*(j)^{1-\sigma_C}}{1-\sigma_C} - \zeta_t^L \frac{l_t(j)^{1+\sigma_L}}{1+\sigma_L} \right], \quad \sigma_C, \sigma_L > 0$$

where ζ_t^b represents a shock to the discount rate and ζ_t^L represents a shock to the labour supply, while the coefficient of relative risk aversion σ_C is also the inverse intertemporal elasticity of substitution, and σ_L represents the inverse Frisch labour supply elasticity; finally, $j \in [0, 1]$ signifies the household. We follow Smets and Wouters (2003) and assume external habit formation in consumption; that is, utility is obtained from

$$C_t^*(j) = C_t(j) - hC_{t-1}, \quad 0 \leq h \leq 1, \quad (2)$$

where hC_{t-1} is the habit stock at time t which is external in the sense that it is proportional to the past aggregate consumption level that is considered exogenous to the individual household. We further assume a security market where households completely diversify their individual income uncertainty, so that consumption is equalised across households; $C_t(j) = C_t, \forall j$.

Each household supplies an idiosyncratic variety of labour service $l_t(j)$. These labour services enter as a Dixit-Stiglitz aggregate in the intermediate-goods firm production; thus, letting $l_t(s, j)$ be the amount of labour service j utilized by firm s we find that firm s uses the following amount of composite labour services;

$$L_t(s) = \left[\int_0^1 l_t(s, j)^{\frac{1}{1+\gamma}} dj \right]^{1+\gamma}, \quad \gamma > 1, \quad (3)$$

where γ turns out to be the net wage markup.

As was the case of intermediary prices, wage setting is staggered a la Calvo (1983). That is, in each period household j only optimizes its wage $w_t(j)$ with probability $1 - D$. The household takes the average wage rate $W_t = \left[\int_0^1 w_t(j)^{-\frac{1}{1+\gamma_t}} dj \right]^{-(1+\gamma_t)}$ as given when it chooses its optimal wage $w_{t,t}$ and will meet any demand for the given type of labour;²

$$l_t(j) = \int_0^1 l_t(s, j) ds. \quad (4)$$

In addition to consumption, households can invest in domestic and foreign one-period bonds as well as in domestic capital. Capital K_t earns rental rate R_t and accumulates as follows with δ

²Note that the optimal wage in any period is identical across households, which is the reason why $w_{t,t}$ can be written without index j .

measuring depreciation;

$$K_{t+1} = K_t (1 - \delta) + I_t - \frac{\Phi (K_{t+1} - K_t)^2}{2 K_t}, \quad 0 < \delta < 1, \quad \Phi > 0, \quad (5)$$

where I_t is investment. Here, we have followed Kollmann (2002) and assumed quadratic adjustment costs. Domestic bonds A_t earns net interest i_t , while the interest i_t^f accruing to foreign bonds B_t held by domestic agents deviates from the exogenously given foreign interest level i_t^* as follows;

$$(1 + i_t^f) = \Omega_t (1 + i_t^*), \quad (6)$$

$$\Omega_t = v_t \exp \left\{ -\lambda \frac{e_t B_{t+1}}{P_t \Xi} \right\}, \quad \Xi = \frac{e P^x Q^x}{P}, \quad (7)$$

where e_t is the nominal exchange rate and P_t is the price of final goods, while Ξ is the steady-state value of export in units of the domestic final good. Thus, the interest on foreign bonds is growing in the foreign debt level which ensures the existence of a unique equilibrium, cf. Schmitt-Grohe and Uribe (2003), while v_t is a stochastic *i.i.d.* shock which we motivate with the empirically observed departure from the uncovered interest parity. We style v_t a UIP shock but abstain from a deeper explanation of its nature; Bergin (2004) offers a good discussion of UIP shocks in the new open-economy macroeconomic (NOEM) literature.

Households own equal shares of domestic firms and thus earn profit from the intermediate-goods firms ($\Delta_t(j)$) in addition to rental rates R_t on the capital, wage income from their labour services and payments from their state-contingent securities ($S_t(j)$). Hence, the budget constraint of household j is

$$\begin{aligned} A_{t+1}(j) + e_t B_{t+1}(j) + P_t (C_t(j) + I_t(j)) = \\ A_t(j) (1 + i_{t-1}) + e_t B_t(j) (1 + i_{t-1}^f) + R_t K_t(j) + \Delta_t(j) + w_t(j) l_t(j) + S_t(j). \end{aligned} \quad (8)$$

Thus, households decide their consumption, wages and investments in accordance with the solution to the following problem;

$$\begin{aligned} \max_{\{C_t(j), A_{t+1}(j), B_{t+1}(j), K_{t+1}(j), w_{t,t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right], \\ \text{s.t. (1)-(8).} \end{aligned}$$

The first-order conditions for domestic and foreign bonds yield regular Euler conditions;

$$(1 + i_t) E_t [\rho_{t,t+1}] = 1, \quad (9)$$

$$(1 + i_t^f) E_t \left[\rho_{t,t+1} \frac{e_{t+1}}{e_t} \right] = 1, \quad (10)$$

$$\rho_{t,\tau} \equiv \beta^\tau (U_{C,\tau} / U_{C,t}) (P_t / P_\tau), \quad U_{C,t} \equiv \frac{\partial U(C_t^*, L_t)}{\partial C_t}, \quad (11)$$

where $\rho_{t,\tau}$ discounts profits at time τ . One should bear in mind, however, that in this case $U_{C,t}$ depends on C_{t-1} as well as C_t due to our assumption of external habits.

Having assumed that the household always meets demand for labour at its chosen wage level, the optimal wage rate at time t is

$$w_{t,t} = \left(\frac{\sum (D\beta)^{\tau-t} E_t \left[\zeta_t^b \zeta_t^L (1+\gamma) W_\tau^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right]}{\sum (D\beta)^{\tau-t} E_t \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right]} \right)^{\frac{\gamma}{\gamma+(1+\gamma)\sigma_L}}.$$

where W_t is the aggregate wage level determined as

$$W_t = \left[D (W_{t-1})^{-\frac{1}{\gamma}} + (1-D) (w_{t,t})^{-\frac{1}{\gamma}} \right]^{-\gamma}.$$

Thus, the infrequent reoptimisation implies that households must consider expectations of all future wage levels and labour supplies when they set their optimal wage.

2.2 Final Goods

Final goods Z_t are produced using intermediate-good bundles from home (Q_t^d) and abroad (Q_t^m) respectively. These intermediary aggregates are combined with a Cobb Douglas technology;

$$Z_t = \left(\frac{Q_t^d}{\alpha^d} \right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m} \right)^{\alpha^m}, \quad \alpha^d + \alpha^m = 1.$$

Each bundle of intermediate goods is a Dixit-Stiglitz aggregate, where ν turns out to be the net markup rate;

$$Q_t^i = \left[\int_0^1 q^i(s)^{\frac{1}{1+\nu}} ds \right]^{1+\nu}, \quad i = d, m.$$

Assuming that domestic firms face the problem of minimizing the cost of producing Z_t units of the final good, demands for goods produced domestically and abroad can be written as

$$\begin{aligned} Q_t^i &= \alpha^i \frac{P_t}{P_t^i} Z_t, \quad i = d, m, \\ P_t &= \left(P_t^d \right)^{\alpha^d} \left(P_t^m \right)^{\alpha^m}, \end{aligned}$$

where the appropriately defined price index P_t is the marginal cost of the final-goods producing firm. With perfect competition in the final-goods market, P_t is also the price of one unit of the final consumption good.

2.3 Intermediate Goods

Intermediate goods are produced from labour L_t and capital K_t using Cobb-Douglas technology. Thus, the production function of firm s is

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1,$$

where θ_t is the exogenously given aggregate level of technology. Producers operate in a monopolistic competitive market, where each producer sets the price of her variety, taking other prices as given and supplying whatever amount is demanded at the price set.

Firms rent capital at the rate R_t and compensate labour with wages W_t . Hence, any firm's marginal costs are

$$MC_t = \frac{1}{\theta_t} W_t^{1-\psi} R_t^\psi \psi^{-\psi} (1-\psi)^{-(1-\psi)}. \quad (12)$$

Producers sell their good variety to both domestic and foreign final-goods producers (that is, $y_t(s) = q_t^d(s) + q_t^m(s)$) and are able to price discriminate between the two markets. As is well-known from the Dixit-Stiglitz models, final-good producers demand individual varieties of intermediaries as follows

$$q_t^i(s) = \left(\frac{p_t^i(s)}{P_t^i} \right)^{-\frac{1+\nu}{\nu}} Q_t^i, \quad i = d, m,$$

and thereby firm profits can be written as

$$\pi^{dx} \left(p_t^d(s), p_t^x(s) \right) = \left(p_t^d(s) - MC_t \right) q_t^d(s) + (e_t p_t^x(s) - MC_t) q_t^x(s).$$

We furthermore assume that foreign exporters produce at unit costs equivalent to the aggregate foreign price level P_t^* and thus generate the following profits in the domestic market;

$$\pi^m(p_t^m(s)) = (p_t^m(s) - e_t P_t^*) \left(\frac{p_t^m(s)}{P_t^m} \right)^{-\frac{1+\nu}{\nu}} Q_t^m.$$

Demands from foreign final-goods producers are assumed to be of the Dixit-Stiglitz form as well;

$$q_t^x(s) = \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x, \quad Q_t^x = \left(\frac{P_t^x}{P_t^*} \right) Y_t^*,$$

where the foreign aggregates P_t^*, Y_t^* are exogenous.

As in the case of wages, we follow Calvo (1983) and assume that a firm only reoptimises its prices in any given period with probability $1 - d$. Given that domestic firms seek to maximise profits discounted with a pricing kernel based on household utility (cf. equation (11)), a firm that reoptimises its domestic price faces the following problem;

$$p_{t,t}^d = \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} \pi^{dx}(\omega, p_t^x(s)) \right],$$

As firms set prices in the domestic and foreign market separately, the constant marginal costs – cf. equation (12) – imply that the two price setting problems are independent. Hence, the optimal price $p_{t,t}^d$ is determined from the following first-order condition;

$$p_{t,t}^d = (1 + \nu) \frac{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{t+\tau}^d MC_{\tau} \right]}{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d \right]}.$$

Import firms are owned by risk-neutral foreigners who discount future profits at the foreign nominal interest rate $R_{t,\tau} \equiv \Pi_{s=t}^{\tau-1} (1 + i_s^*)^{-1}$. Thus, they set their prices in order to maximize discounted future profits measured in foreign currency;

$$p_{t,t}^m = \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [R_{t,\tau} \pi^m(\omega) / e_{\tau}]$$

which again implies a condition for the optimal price $p_{t,t}^m$ similar to that for $p_{t,t}^d$.

Finally, the aggregate Dixit-Stiglitz prices of the intermediate goods are as follows;

$$P_t^i = \left[d (P_{t-1}^i)^{-\frac{1}{\nu}} + (1 - d) (p_{t,t}^i)^{-\frac{1}{\nu}} \right]^{-\nu}, \quad i = d, m, x.$$

2.4 Market Clearing Conditions

All intermediaries are demanded from either domestic or foreign final goods producers, while final goods can either be consumed or invested in capital. Hence, equilibria in the markets for intermediate and final goods require

$$\begin{aligned} Y_t &= Q_t^d + Q_t^x, \\ Z_t &= C_t + I_t. \end{aligned} \tag{13}$$

Turning to the capital market, aggregate demand for capital is

$$\mathcal{K}_t = \int_0^1 K_t(s) ds = \frac{1}{\theta_t} \left(\frac{\psi}{1 - \psi} \frac{W_t}{R_t} \right)^{1-\psi} \left[q_t^d(s) + q_t^x(s) \right],$$

and, hence, equilibrium in the capital market ($K_t = \mathcal{K}_t$) implies

$$K_t = \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{W_t}{R_t} \right)^{1-\psi} \left[\left(\frac{\mathcal{P}_t^d}{P_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{\mathcal{P}_t^x}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right],$$

where we introduce

$$\mathcal{P}_t^i \equiv \left[\int_0^1 (p_t^i)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad i = d, x.$$

Under the assumptions of the Calvo pricing model, these indices of individual prices evolve as follows;

$$\mathcal{P}_t^i = \left[d (\mathcal{P}_{t-1}^i)^{-\frac{1+\nu}{\nu}} + (1-d) (p_{t,t}^i)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad i = d, x.$$

Finally, we assume that only domestic agents hold the domestic bond, implying that $A_t = 0$ in equilibrium.

Aggregating and manipulating the household budget constraint (8) and using the final-good market equilibrium (13) yields the following equation which simply states that the net foreign assets position (NFA) changes with accruing interest and the net export.

$$\begin{aligned} e_t B_{t+1} + P_t (C_t + I_t) &= e_t B_t \left(1 + i_{t-1}^f \right) + R_t K_t + W_t L_t \\ &\quad + P_t^d Q_t^d + e_t P_t^x Q_t^x - (R_t K_t + W_t L_t) \Rightarrow \\ B_{t+1} &= B_t \left(1 + i_{t-1}^f \right) + P_t^x Q_t^x - \frac{P_t^m}{e_t} Q_t^m. \end{aligned}$$

2.5 Monetary Policy

We have two monetary policy regimes to consider. The first one is a peg regime, as presented in Dam and Linaa (2005), and the second one is a regime in which the central bank conducts monetary policy according to a Taylor rule, first suggested by Taylor (1993) and thoroughly discussed in, e.g., Woodford (2003).

With regards to the peg regime, we postulate that it is impossible for the central bank to keep the exchange rate fully fixed. This is motivated from noting that although the Danish central bank successfully has been able to keep the Danish krone stable vis-a-vis its anchor, minor movements in the exchange rate of first D-mark and then (to a lesser extent) the euro has occurred. Hence, we assume that the central bank can keep the exchange rate fixed around its parity (equal to the steady state value) up to a multiplicative exogenous policy shock ξ_t^{peg} with unity mean;

$$e_t = e \xi_t^{peg}.$$

We assume that $\xi_t^{peg} = \varrho^m \xi_{t-1}^{peg} + \varepsilon_{peg,t}^m$ where $\varepsilon_{peg,t}^m$ is a Gaussian innovation with mean 0 and standard deviation σ_{peg}^m , and $0 \leq \varrho^m < 1$ is the policy error autocorrelation. The intuition of this policy is clearest if we combine it with the Euler equations (9)-(10) and the equations (6)-(7)

describing the wedge on the international interest rate and perform a log-linearisation around the steady state. Then we obtain the following equation for the domestic interest rate;

$$\hat{i}_t = \hat{i}_t^* + \left(\hat{v}_t - \lambda \hat{B}_t \right) + E_t \Delta \hat{\xi}_{t+1}^{peg},$$

where hats indicate a relative deviations from the steady state with proper normalisations.³ Thus, the interest rate responds (virtually) one-to-one with the foreign interest rate and the UIP shock. Furthermore, a positive spread between the foreign and domestic interest rates emerges as the net foreign position of the domestic country becomes negative *et vice versa*. Besides being intuitively appealing, the debt premium also ensures the existence of a unique deterministic steady state.

The alternative Taylor rule is discussed in Section 5 below.

3 Parameterisation

To perform a quantitative welfare analysis and to produce impulse-response functions we need to assign values to the parameters in the model. In Dam and Linaa (2005) we estimated the model using a Bayesian estimation technique; that is, we used the Kalman filter to evaluate the likelihood of a log-linearised version of the model and combined that information with our prior assumptions on the structural parameters in order to obtain the posterior estimates. However, since we had to have the markup rates as constants in this analysis as discussed above, we necessarily have to deviate from the estimation results obtained in that paper. Before justifying the values chosen for the parameters we begin by summarising the parameterisation in Table 1.

Regarding preferences, these posterior estimates imply a labor supply (Frisch) elasticity of approximately one and an intertemporal elasticity of substitution of a half. Thus, our labour supply elasticity is in accordance with a rich body of microeconomic findings, yet in the lower range of the values typically used in the RBC literature. The estimate for the external habit stock h lies between one third and a half; this is on the lower side compared with the literature at large, but should be uncontroversial.

The estimated Calvo parameters imply that prices and wages are updated every four years and one year, respectively. While the latter is plausible, the former implies an implausibly high degree of price rigidity. We discuss potential causes of this puzzling finding in Dam and Linaa (2005) and we return to its implication for the welfare analysis in Section 7.

As mentioned, a difference between the current peg model and the model presented in Dam and Linaa (2005) is the absence of stochastic movements in the mark-up rates in this paper. Hence, we have fixed γ and ν at the values used as means in the markup processes in Dam and Linaa (2005). In that paper we also obtained a value of σ_L which was very high; we have thus attached a new value to this parameter based on obtaining a predicted standard deviation of Y_t (in the peg model) that approximately matches that of its empirical counterpart, GDP.

³We refer the reader to Dam and Linaa (2005) for the exact details of a log-linearisation of the model.

Table 1: Parameter Values

Parameter		Value
α^d	Share of domestic intermediaries in final prod.	0.7
β	Discount factor	0.99
δ	Depreciation rate	0.025
ψ	Capital share	0.33
λ	Capital mobility parameter	0.0019
d	Calvo, intermediaries	0.941
D	Calvo, wages	0.770
h	Habit persistence	0.433
σ_C	Household inverse IES	1.709
σ_L	Inverse Frisch elasticity	1.032
Φ	Capital adj. cost	14.422
ν	Wage markup	0.2
γ	Price markup	0.2
<i>Shocks, persistence</i>		
ϱ^b	Discount rate	0.825
ϱ^l	Labor supply	0.962
ϱ^t	Technology	0.824
ϱ^m	Peg	0.899
ϱ^i	Foreign interest rate	0.877
ϱ^P	Foreign price level	0.925
ϱ^Y	Foreign GDP	0.912
<i>Shocks, volatility</i>		
σ^b	Discount rate	0.041
σ^l	Labor supply	0.0295
$\sigma^t \times 100$	Technology	1.073
$\sigma^U \times 100$	UIP	0.342
$\sigma_{peg}^m \times 100$	Peg	0.739
σ_{TR}^m	Monetary policy shock (Taylor rule)	0.08
$\sigma^i \times 100$	Foreign int. rate	0.102
$\sigma^P \times 100$	Foreign price level	0.337
$\sigma^Y \times 100$	Foreign GDP	0.786

Apart from this, and apart from the values of $\alpha^d, \beta, \delta, \psi, \lambda$ and η which were kept fixed in the estimation of the model in Dam and Linaa (2005), we use the values obtained as modes in our posterior distribution.

4 Welfare Measure and Solution Method

Our measure of welfare is the unconditional expectation of household utility;

$$E \left[\int_0^1 U \left(C_t^k - hC_{t-1}^k, l_t^k(j) \right) dj \right],$$

where k refers to the particular policy rule. As discussed thoroughly in Kim et al. (2003), this amounts to comparing welfare in the different stochastic steady states associated with each monetary policy rule under consideration; hence, this measure implicitly disregards any welfare effects stemming from the transition between the initial state of the economy and the stochastic steady state under the considered rule.

Integrating utility over the households is unproblematic with respect to consumption as we have assumed a security market that equates consumption across them, cf. Subsection 2.1. Labour supply, however, has not been smoothed between the households, and thus we need to pay attention to the integral of the disutility of labour. Integrating over the disutility yields

$$\int_0^1 l_t(j)^{1+\sigma_L} dj = L_t^{1+\sigma_L} \left(\frac{\mathcal{W}_t}{W_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)},$$

where

$$\mathcal{W}_t \equiv \left[\int_0^1 w_t(j)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} dj \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}.$$

Due to the assumptions of the Calvo-like wage setting, this index of wage dispersion evolves as follows;

$$\mathcal{W}_t = \left[D\mathcal{W}_{t-1}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} + (1-D) w_{t,t}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}.$$

Thus, the welfare measure can be cast as follows;

$$E \left[\int_0^1 U \left(C_t^k - hC_{t-1}^k, l_t^k(j) \right) dj \right] = \frac{\zeta_t^b}{1-\sigma_C} (C_t - h\bar{C}_{t-1})^{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} L_t^{1+\sigma_L} \left(\frac{\mathcal{W}_t}{W_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)}.$$

Given the complexity of our non-linear model, an analytical solution is unattainable. Instead, we obtain a second-order approximation with the DYNARE program.⁴ We have chosen this solution method since first-order approximations are not adequate for welfare analysis of stochastic models. We refer the reader to Kim and Kim (2003) for an example of the inadequacy of first-order

⁴The DYNARE program is an ongoing project at CEPREMAP and can be downloaded at www.ceprenap.cnrs.fr/dynare/ where documentation is also available.

approximations, and to Schmitt-Grohe and Uribe (2004b) for a thorough discussion of the merits of second-order approximations.

Application of the DYNARE solution method requires that we write our model in the following general form;

$$E_t [\Upsilon_t, \Upsilon_{t+1}, \varepsilon_t, \varepsilon_{t+1}] = 0, \quad (14)$$

where Υ_t is a vector of the endogenous variables of the model, while ε_t is a vector containing the innovations to the structural shock processes.⁵ Thus we recast the model in the iterative form of (14) where we also normalise all nominal variables with the price of domestic (or foreign) final goods. The normalisation is carried out since the Taylor rule will only pin down the inflation rate, not the price level, and we want to work with a stationary system. This version of the model is summarised in Appendix A.⁶

5 Finding the Optimal Taylor Rule

Our alternative to the existing peg is an independent monetary policy rule belonging to the generalised family of Taylor rules. In particular, we restrict ourselves to the following variant of the interest rule;

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left(\rho_\pi (\Pi_t - 1) + \rho_y \left(\frac{Y_t}{Y_{t-1}} - 1 \right) \right) + \xi_t^{TR},$$

where the ρ 's are the policy parameters which should be optimised to the economy in question, while ξ_t^{TR} is a Gaussian *i.i.d.* noise term reflecting monetary policy shocks. The standard deviation of this shock cannot be estimated on Danish data since this monetary regime has never existed. Instead the parameter σ_{TR}^m is attached a value equal to 0.0008, which is the posterior mode estimate found by Smets and Wouters (2003) on data for the euro area. We return to this issue in Section 7 below.

We perform a grid search of the policy parameters in the ranges $\rho_i \in [0; 0.9]$, $\rho_\pi \in [0; 3]$ and $\rho_y \in [0; 3]$. We consider increments of 0.15 for the smoothing parameter ρ_i which is usually introduced in order to capture empirically observed policy inertia; we include it in this normative exercise, however, since smoothing can improve welfare in some cases as shown by Schmitt-Grohe and Uribe (2004b). For ρ_π and ρ_y we consider increments of 0.10. Thus, we solve the model for 6727 different configurations of the Taylor rule.

Schmitt-Grohe and Uribe (2004a) formulate three requirements to what they style *operational rules*; they must (i) respond only to a limited set of readily observed variables; (ii) induce a locally unique rational-expectations equilibrium; and (iii) satisfy the non-negativity constraint on nominal interest rates. The first requirement is clearly fulfilled, as we only consider observed variables in the rule in the form of realised levels or growth rates of overall inflation, GDP (and the nominal exchange rate). In light of the controversy regarding the actual calculation of output gaps, we find

⁵See Schmitt-Grohe and Uribe (2004c) for a presentation and derivation of the solution method we apply through DYNARE.

⁶The transformation of the nominal model to real terms is documented in the technical appendix.

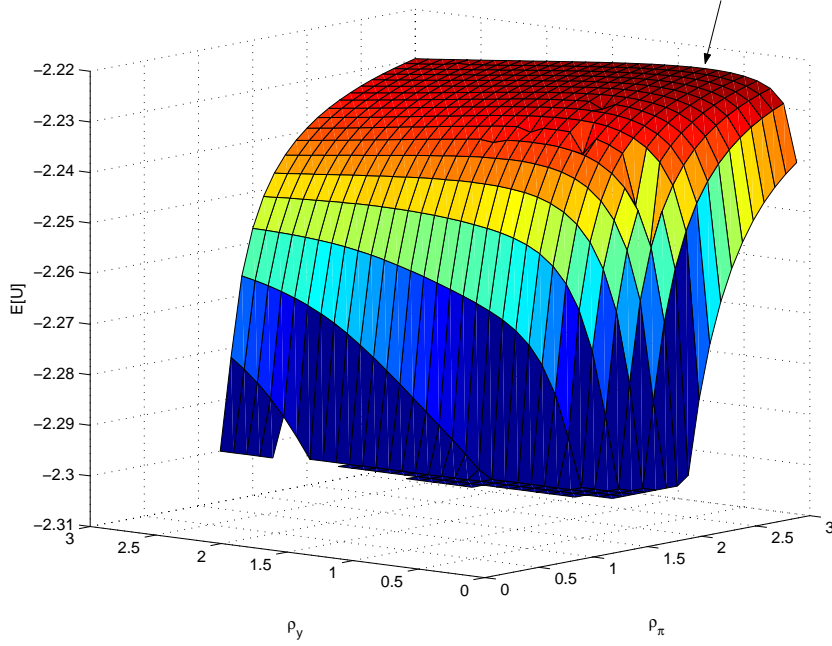


Figure 1: Unconditional utility as a function of the Taylor rule parameters

that this restriction on the functional form of the rule is justified.⁷ To meet the second requirement we only consider configurations of the rule that yield a determinate equilibrium in a radius of 0.2 of the parameters under consideration. This is done in order to avoid configurations close to bifurcation points which tend to invalidate the welfare calculations, cf. Schmitt-Grohe and Uribe (2004b). Thirdly, we follow Schmitt-Grohe and Uribe (2004a) and formulate the non-negativity constraint indirectly through a condition that unconditional expectation of the interest rate should be greater than twice its standard deviation ($E[i_t] > 2\sigma_{i_t}$). This requirement is fulfilled for both the peg and the preferred Taylor rule regime.

We first consider the simple version of the Taylor rule, that is, one with no interest smoothing ($\rho_i = 0$). The unconditional utility is shown as a function of the two policy rule parameters in Figure 1.⁸ The maximum utility is obtained for the configuration $(\rho_\pi, \rho_y) = (3, 0.8)$.

Interestingly, the optimal rule does not change when we introduce interest smoothing. That is, unconditional utility is maximised at $(\rho_\pi, \rho_y, \rho_i) = (3, 0.8, 0)$ which is illustrated in Figure 2.⁹ Hence, this rule will be the preferred one in the following section where we compare its merits with

⁷Here it could be argued that the central bank does not have information on Y_t at time t when it is to choose i_t . We acknowledge that, but defend our choice by claiming that the central bank should have a relatively reliable forecast regarding Y_t at time t .

⁸Configurations of the policy rule that implies a determinate equilibrium with $E[U] < -2.3$ are assigned that value in Figure 1 for instructive purposes. The calculations have suffered from numerical problems which we have not been able to resolve. Thus, a few of the points have been obtained from interpolation from neighbourhood points within an 0.03 radius. Details are available from the authors upon request.

⁹Configurations of the policy rule that implies a determinate equilibrium with $E[U] < -2.25$ are assigned that value in Figure 2 for instructive purposes. The remarks on interpolation in Footnote 8 also applies here.

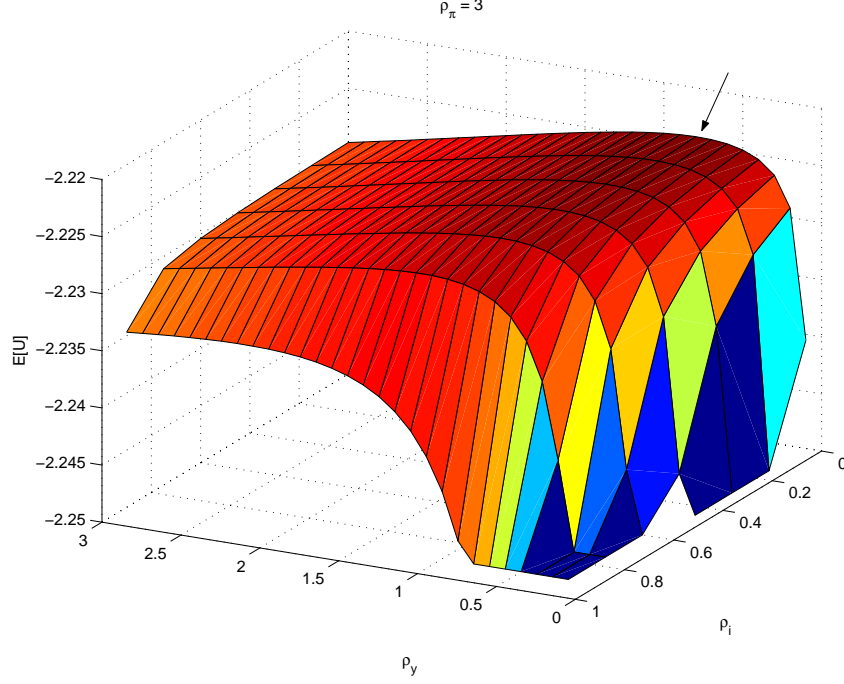


Figure 2: Unconditional utility and interest rate smoothing

those of a fixed exchange rate.

Woodford (2003) establishes the optimality of a Taylor rule in a model similar in spirit to the one we have formulated. However, the optimality requires an output gap measure in the rule based on an economy with no nominal rigidities, while inefficiencies in the economy are assumed to have been eliminated through taxes and subsidies. Hence, optimality of our Taylor rule is unlikely in a wider sense, and thus the welfare gains which we find from an independent monetary policy compared with the existing peg regime only constitute a lower bound on the gains that could be obtained. We do, however, believe that the familiarity and straightforward operationality of the rules we consider is in itself an asset that motivates interest in this particular choice of monetary policy.

6 Results

In this section we analyse the welfare implication of the two monetary policy regimes under consideration as well as their causes.

6.1 Welfare

We measure the welfare gain of a Taylor rule over the existing peg regime through compensating variation. That is, we calculate the relative permanent change in consumption that equates the unconditional utility of households under the peg regime with that obtained under the optimal

Taylor rule. Thus, the compensating variation of consumption is defined as the χ that solves the following equation;

$$E \left[\int_0^1 U (C_t^{TR} - hC_{t-1}^{TR}, l_t^{TR}(j)) dj \right] = E \left[\int_0^1 U ((1 + \chi) (C_t^{peg} - hC_{t-1}^{peg}), l_t^{peg}(j)) dj \right].$$

Table 2: Welfare Analysis

Std. deviations (in pct)	<i>Peg</i>	<i>Taylor</i>
<i>Y</i>	2.85	3.36
<i>C</i>	3.22	4.99
<i>I</i>	7.03	10.75
<i>L</i>	4.25	4.09
<i>i</i>	0.44	0.19
Π	0.11	0.07
Means (in pct)		
<i>Y</i>	-0.64	-0.25
<i>C</i>	-1.16	-0.44
<i>I</i>	-1.26	0.06
<i>L</i>	-0.11	-0.08
<i>i</i>	-0.01	0.00
Π	0.00	0.00
Welfare equiv. χ (pct of <i>C</i>)		0.792
Note: All reported statistics are relative deviations from the non-stochastic steady state.		

We see from Table 2 that moving from a peg regime to one where the monetary policy is set according to a Taylor rule results in a welfare improvement of 0.79 pct measured in units of consumption goods. On the one hand, we note that although consumption is more volatile under the Taylor regime compared with the peg, the level of consumption has increased. With regards to labour supply this result is reverted; labour supply is more volatile under the peg than in the Taylor rule regime, while the mean of labour supply is lower under the peg. Overall, the household prefers the higher consumption under the Taylor regime even though they need to work more in order to obtain this.

Contrary to Kollmann (2002) we find that volatility in output is higher under a Taylor rule than under the peg. This observation is attributed the existence of the highly persistent labour supply shock we consider in this paper. Decomposing the contribution from the shocks reveals the findings reported in Table 3. As stressed by Dam and Linaa (2005), labour supply shocks are the overall dominant source of fluctuations. To verify that this is indeed the main reason behind the increased volatility of the Taylor rule regime compared with the peg, we ran both simulations under the assumption that $\varrho^L = 0.82$ which is the autocorrelation estimated for the technology process. In this case we obtained a standard deviation of output equaling 1.81 pct in the peg regime dropping substantially to 1.00 pct under the Taylor regime, thus re-establishing the findings of Kollmann (2002). In this scenario the welfare gain by leaving the peg and adopting a Taylor rule dropped to

0.28 pct measured in units of consumption goods.

Table 3: Variance Decomposition

Variable		Y	C	I	L	i	Π
Preferences	Peg	0.33	0.90	0.16	0.29	0.02	0.05
	Taylor	0.05	0.28	1.26	0.04	1.54	1.71
Labour supply	Peg	61.17	78.41	42.35	31.79	0.10	75.49
	Taylor	93.97	90.16	81.47	56.31	55.70	48.03
Technology	Peg	2.41	1.33	2.37	39.05	0.00	11.51
	Taylor	0.54	0.15	0.60	41.83	34.18	41.19
UIP	Peg	0.41	0.14	1.56	0.42	62.04	0.02
	Taylor	0.00	0.00	0.00	0.00	0.00	0.00
Monetary policy	Peg	13.57	11.72	28.89	10.20	15.73	0.43
	Taylor	1.57	1.21	2.82	0.98	1.97	2.10
Foreign interest rate	Peg	5.29	3.63	12.85	4.54	21.31	1.97
	Taylor	0.18	0.14	0.44	0.12	0.50	0.46
Foreign price	Peg	2.95	1.12	2.41	2.39	0.15	0.73
	Taylor	0.06	0.17	0.96	0.09	0.86	0.86
Foreign demand	Peg	13.86	2.75	9.41	11.34	0.65	9.79
	Taylor	3.63	7.88	12.46	0.63	5.24	6.65
Note: All shares are in pct.							

In Figure 3 we compare the unconditional utility as a function of the Taylor parameters (ρ_π, ρ_y) with that obtained under the peg. We see that a rather large set of parameters of ρ_π and ρ_y ensures a level of utility that exceeds the level of utility under the peg.

6.2 Impulse-Response Functions

This section clarifies the important deviations between the economy in which monetary policy is conducted according to a Taylor rule and one in which a constant nominal exchange rate is the monetary policy target. In particular we seek to clarify why volatility in consumption is higher under a Taylor rule than in the peg regime and why volatility in labour is lower. We do so by studying the impulse-responses obtained from both models. Inspecting the consequences of a technological shock, we see from Figure 4 that under the peg output initially drops. This phenomenon was thoroughly analysed in Dam and Linaa (2005); the initial drop in output is a consequence of the very rigid prices; recall the Calvo parameter in the intermediary sector is estimated to be as high as 0.94. Thus, even though the positive shock to technology shifts the supply curve of the firms to the right, the price inertia causes the short-run supply curve to be almost horizontal, and thus the direct supply-side effect on output is small. Furthermore, a given level of production can now be reached

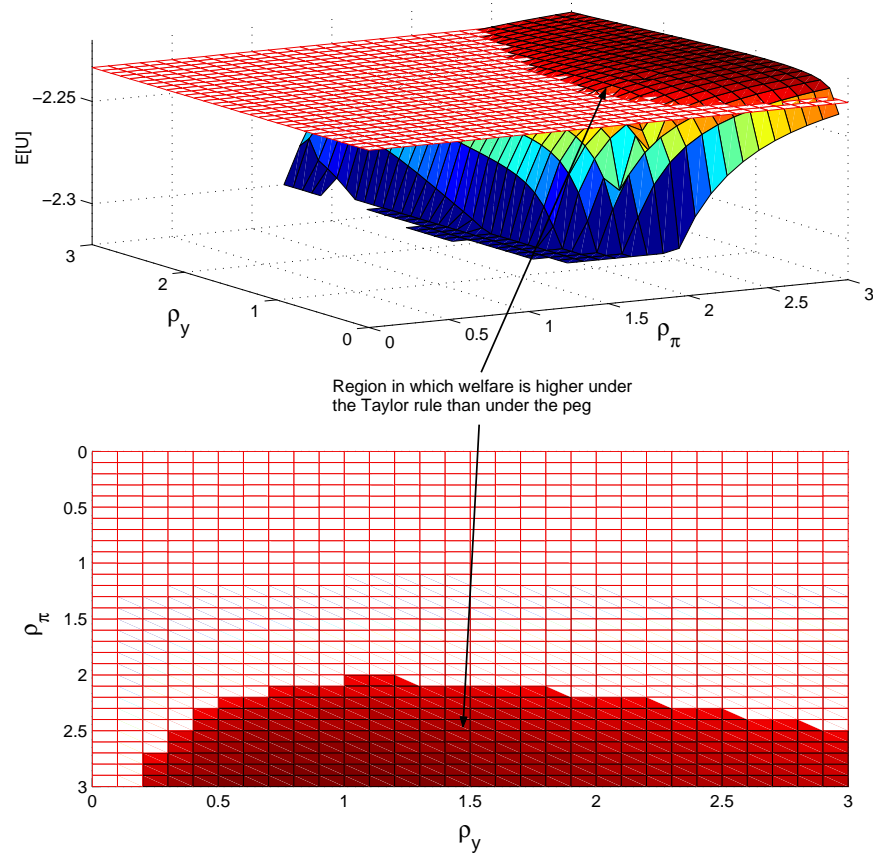


Figure 3: Unconditional utility under the peg and the Taylor rule, resp.

using fewer production resources due to the higher level of productivity, causing employment as well as capital demand to decrease. In turn, households wish to hold less capital stock and disinvest. Thus, total demand for final goods has fallen, and in equilibrium this effect dominates the positive supply effect, implying a lower output equilibrium than before the shock. Over time, however, prices do fall because of the persistent technology shock that has decreased marginal costs, and as demand responds to the lower prices, capital is accumulated and investments rise. A crucial difference between the peg and the Taylor regime is the central bank's reaction to such a shock; under the peg the central bank keeps the interest rate virtually at the pre-shock level because the exchange rate is nearly unaffected by the shock. Over time, however, domestic prices fall since fewer resources are required to produce a given amount of goods; this drop in inflation trickers the central bank under the Taylor regime to lower interest rates. While the response of investments in the models is almost the same, we see that under a Taylor rule consumption initially benefits from the lower interest rates (as the return of holding bonds has declined), thereby bringing total demand into the positive region, ensuring a positive initial response in output.

In Figure 5 we inspect the consequences of an expansionary labour supply shock. The shock represents a shift in the household's relative valuation of consuming and enjoying leisure. Again we observe that under the peg, output initially drops for the same reasons as stated for the technology shock. Responses in consumption to a labour supply shock are far more persistent than the responses following a technology shock are for two reasons; first, the labour supply shock in itself is more persistent than the technology shock is, and second, the labour supply shock changes the relative valuation of consumption relative to leisure. For the same reasons, we also see that persistence in labour responses increase compared to those of a technological shock.

Summarising, we found that three shocks are of great importance for the volatility in labour supply; technology, monetary policy and labour supply shocks. While the response stemming from technological shocks are almost identical in the two models, we just saw that labour supply shocks contribute to generating an aggregated level of volatility in consumption and labour that is higher under a Taylor rule than in the peg regime.

This is reverted, however, when studying expansive shocks to monetary policy, cf. Figure 6. Under the peg, this experiment corresponds technically to shocking ξ_t^{peg} , thereby devaluating e_t . Since this shock is autocorrelated, e_t will remain undervalued compared to its parity for periods to follow. The lower level of interest rates stimulates consumption as well as investments and output rises. Under the Taylor rule the experiment is slightly different; ξ_t^{TR} is negatively shocked, and initially this is expected to induce a fall in i_t . However, when the central bank lowers interest rates households prefer to consume or hold foreign bonds instead of domestic ones; the first effect causes output to increase while the latter effect puts pressure on the exchange rate and inflation rises. The degree of price stickiness is very high, and therefore the central bank immediately reacts by hiking interest rates since also distant future periods is weighted heavily. Additionally, the increase in output causes the central bank to contract monetary policy; this endogenous response in interest rates is larger than the exogenous response stemming from the shocks is, and therefore

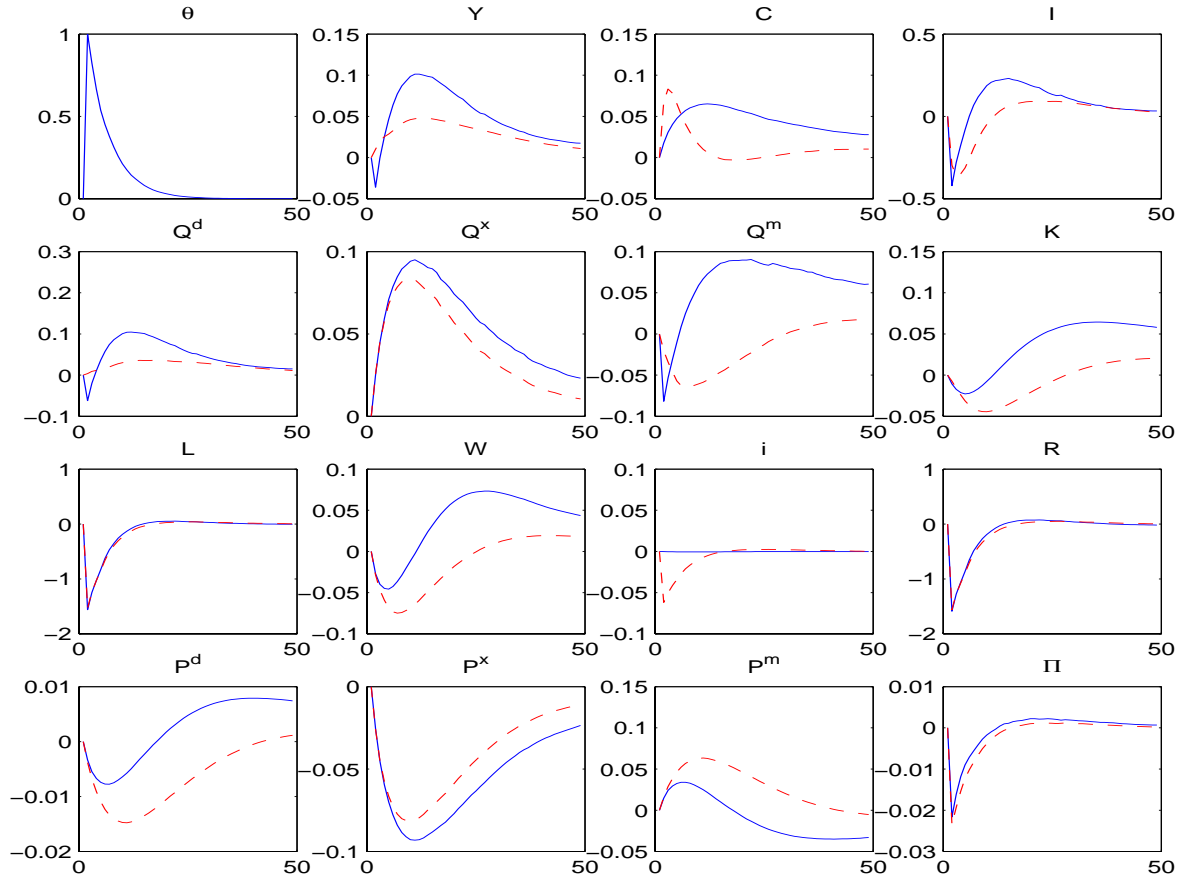


Figure 4: Responses to a technology shock (Peg: Solid lines, Taylor: Dashed lines)

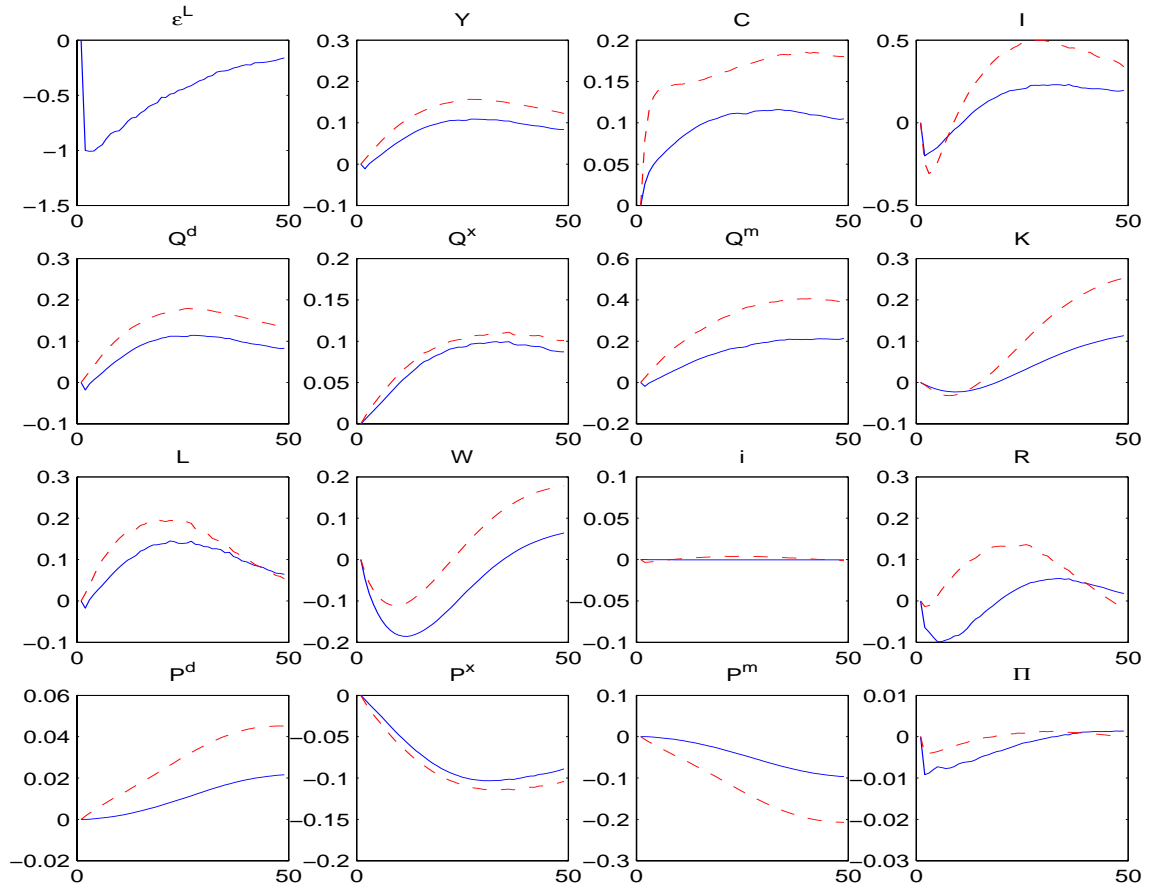


Figure 5: Responses to a labour supply shock. (Peg: Solid lines, Taylor: Dashed lines)

our experiment of performing an expansionary monetary policy shock results in initially rising interest rates. For the same reason the responses in consumption and investments, and hence output, are more muted than under the peg. In the short term, however, the economy benefits from the expansionary effects stemming from a devaluated exchange rate that is higher than under the peg. We finally note that volatility in labour is higher under the peg than in the Taylor rule regime, thereby contributing to the finding that labour is more volatile under the peg.

Finally, in Figure 7 we observe what happens following an exogenous shock to foreign interest rates. Under the peg, the domestic central bank has to follow the direction of the foreign interest rate movements in order to keep the exchange rate fixed. In the Taylor rule regime the central bank hardly reacts as inflation as well as output is unaffected initially by the foreign interest rate shock. For the same reason, only minor movements of all variables are observed in this case. Under the peg, however, the higher level of interest rates dampens consumption as well as investments causing aggregate output to fall. Again, we note that labour is more volatile under the peg, but movements in the foreign interest rate are of less importance for movements in labour than are technology, labour supply and monetary policy shocks.

7 Robustness and Alternative Scenarios

The previous section demonstrated that there are welfare gains from changing the monetary policy from a fixed exchange rate to a Taylor rule. Recall, that the values of σ_{peg}^m and σ_{TR}^m quantify the volatility of the policy errors under the two regimes. Considering the nature and scope of the deregulated foreign-exchange market of today, one should generally find that the central bank's task of assigning the interest rate level that keeps the exchange rate exactly on target is nontrivial, and thus a certain amount of policy errors seems unavoidable. Administering a Taylor rule with fixed intervention dates and infrequent observations of the inflation and output gaps seems like a manageable task in comparison. However, if it is possible for a central bank to obtain a credible peg on a foreign currency, pressure on the exchange rate could plausibly fall to a level where the peg can be maintained with a degree of precision comparable to that of a Taylor rule. Indeed, the recent Danish experience has been one of a very stable exchange rate around the fixed parity, as is evident from Figure 8. It turns out that varying the volatility of the policy shock in the range spanned by the Taylor rule estimate of Smets and Wouters (2003) and that obtained for the Danish peg regime in Dam and Linaa (2005) is of critical importance for the welfare results.

As described in Section 3 we are unable to estimate the volatility of ξ_t^{TR} since this regime has not been in effect in Denmark. Instead we relied on the estimated volatility obtained by Smets and Wouters (2003) as a proxy for “what to expect” if this monetary policy regime was introduced in Denmark. In Figure 9 we show the welfare equivalences χ for varying values of σ_{TR}^m . For a value of σ_{TR}^m slightly above 0.005 we see that this compensation becomes negative meaning that if policy errors under the Taylor rule lies above this level, a regime shift in monetary policy would result in a welfare loss.

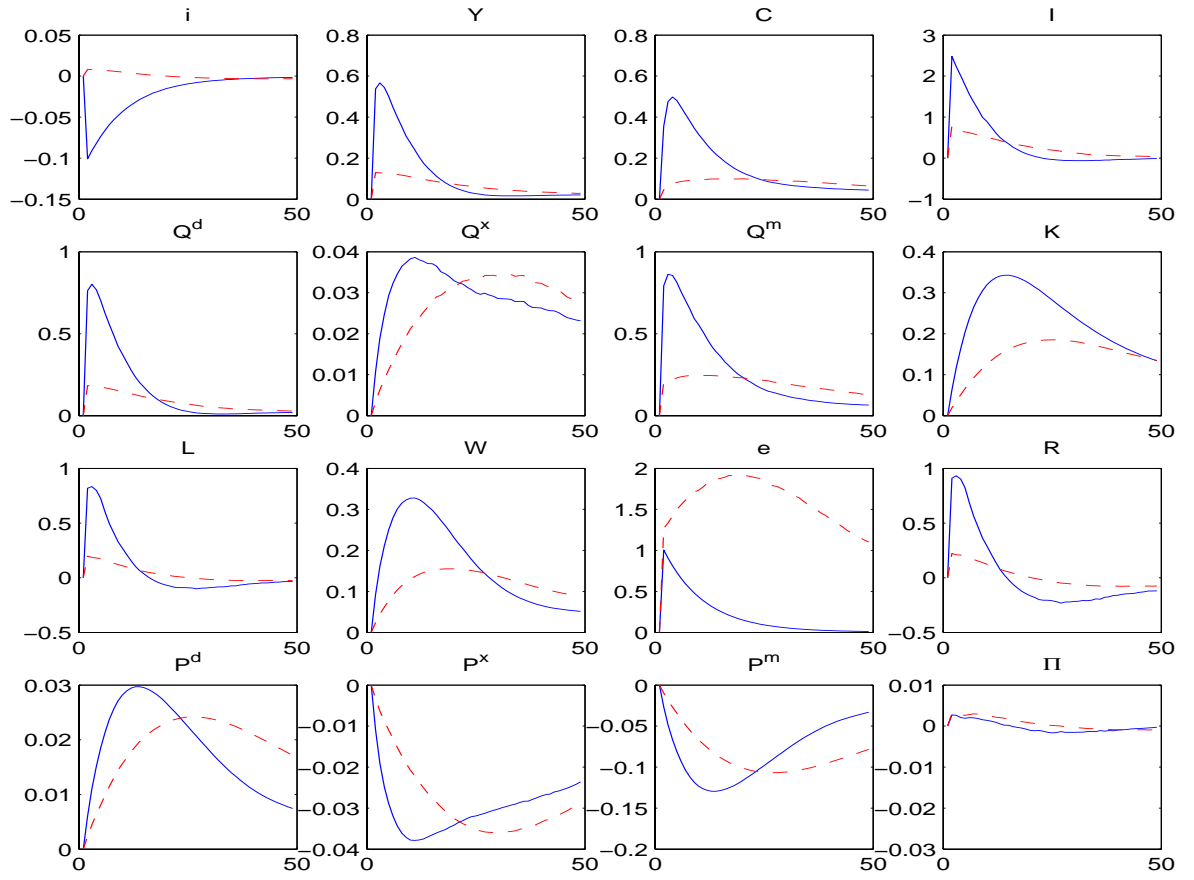


Figure 6: Responses to a monetary policy shock. (Peg: Solid lines, Taylor: Dashed lines)

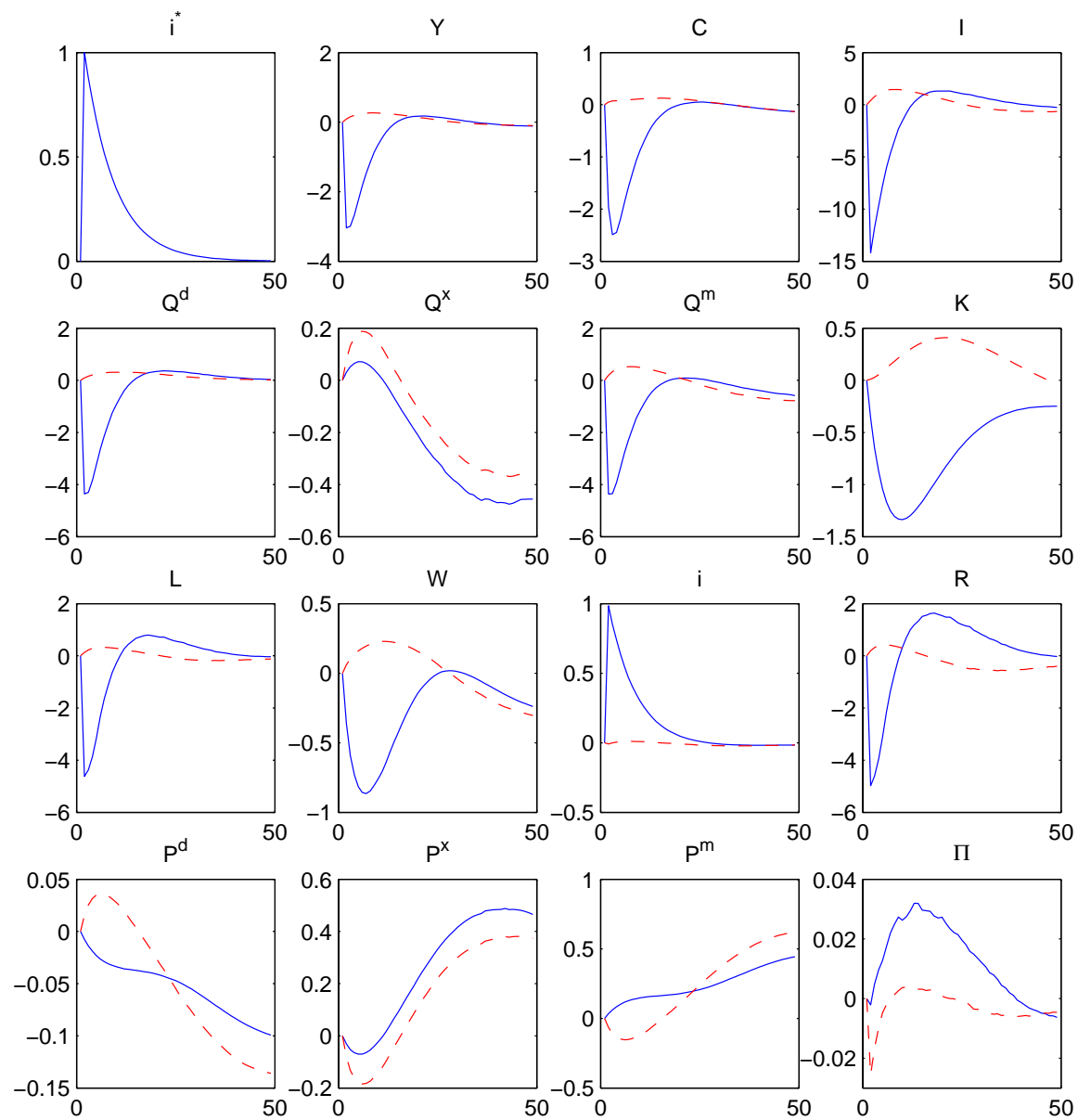


Figure 7: Responses to a foreign interest rate shock. (Peg: Solid lines, Taylor: Dashed lines)

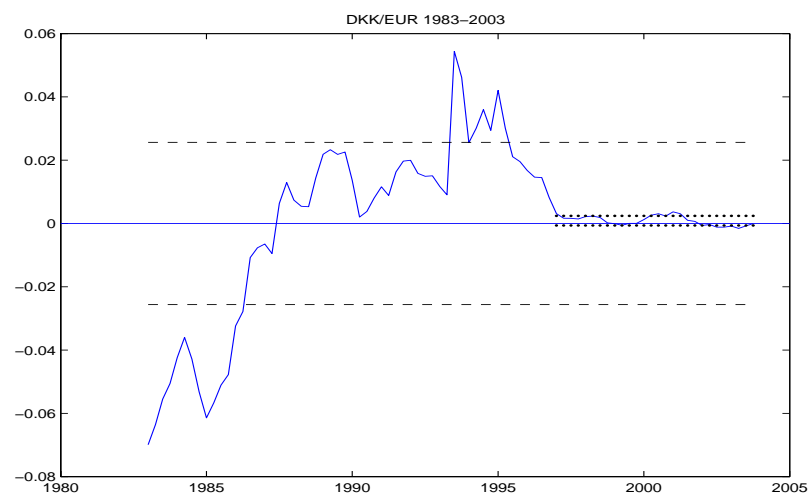


Figure 8: Log of DKK/EUR 1983-2003. Linearly detrended. Dashed and dotted lines are standard deviations for the entire sample and 1997-2003, resp.

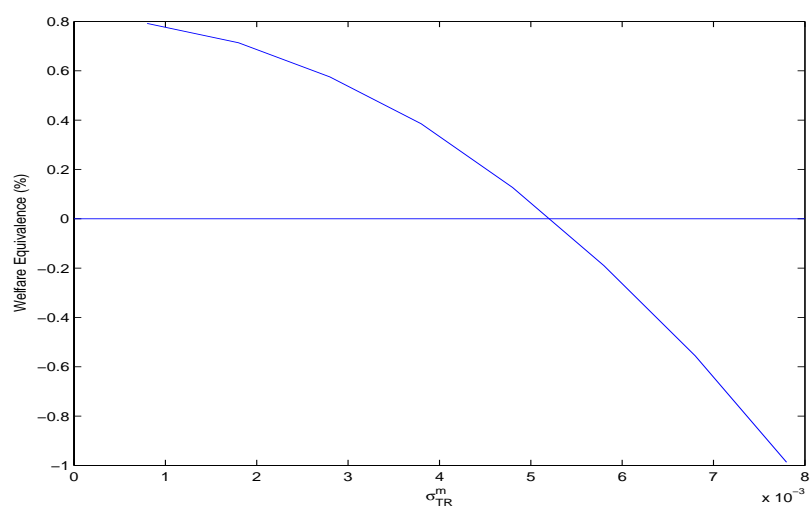


Figure 9: Welfare equivalence and monetary policy volatility

We also turned this experiment on its head; we estimated an AR(1) for the exchange rate since 1999. As was seen from Figure 8, volatility in this period has been substantially reduced compared to that of the full sample. This estimation resulted in an autocorrelation, ϱ^m , equal to 0.86, while σ_{peg}^m was estimated to the value of 0.0008, equal to the value of σ_{TR}^m . Working with this process decreased the benefits of adopting the Taylor rule, although it was still advisable, cf. Scenario I in Table 4.

We also carried out two additional simulations of alternative scenarios as is seen from Table 4. Scenario II has already been described in Section 6 and it took the form of assuming labour supply shocks were no more persistent than technology shocks. In this case ϱ^L was attached a value of 0.82 (equal to ϱ^t) and this reduced the compensation in consumption needed to put household utility under the peg equal to utility in the Taylor rule regime to 0.27 pct.

Table 4: Welfare Analysis - Alternative Scenarios

<i>Scenario</i>		$100 \times \chi$
I	Lower exchange rate volatility ($\sigma_{peg}^m = 0.0008$, $\varrho^m = 0.86$)	0.660
II	Less persistent labour supply shock ($\varrho^L = 0.82$)	0.277
III	Less nominal rigidity ($d = D = 0.75$)	1.149
Note: χ measures the compensating variation of consumption between the peg and the optimal Taylor regime as defined in equation (??).		

Scenario III was assuming that prices were less rigid than they were estimated to be; postulating both goods prices as well as wages can be reoptimised once a year increases the compensation that equalises welfare between the two regimes to 1.15 pct. This is the result of two opposing sources; on the one hand, when prices are extremely rigid, an inflation fighting central bank has only limited possibilities to control inflation. This tends to reduce the benefits from leaving the peg and adopt the Taylor rule. On the other hand, however, damages by not controlling inflation are more severe, since they last longer. In this case the first source dominate.

The overall conclusion therefore seems to be that Denmark could potentially benefit from giving up the peg and begin conducting monetary policy according to a Taylor rule. This change in the monetary policy regime seems to be beneficial unless the policy error, σ_{TR}^m , takes a substantially larger value, than Smets and Wouters (2003) estimated as the value relevant for the euro-zone.

8 Conclusion

In this paper we analysed the consequences of Denmark replacing the peg with a Taylor rule. For this purpose we used the model laid out and estimated in Dam and Linaa (2005) in order to quantify the welfare implications in the two regimes. We then dropped the assumption of the central bank following a peg and replaced it with an assumption of the central bank conducting monetary policy according to a Taylor rule.

The models tell us that it is possible to increase the level of welfare by doing so; in fact we find that the benefits can be summarised to 0.79 pct. measured in units of consumption goods. Various alternative scenarios did not change this conclusion although the magnitude of change in welfare, of course, was affected by this. It turned out that welfare under the peg would only exceed welfare in the Taylor regime if policy errors in the Taylor regime are far larger than those estimated for the euro-zone by Smets and Wouters (2003).

Contrary to the related study in Kollmann (2002) we find that volatility of both consumption and output increases when going from a peg to the Taylor rule; the main explanation for this was the existence of a highly volatile and persistent labour supply shock. Reducing the persistence of this shock puts us back to Kollmann's scenario in which volatility is lower in the Taylor regime.

There are, however, potentially important matters not included in the above mentioned framework. If Denmark decided to adopt a Taylor rule, risk aversion from foreign investors might induce a reduction in direct investment flows into Denmark caused by an increased uncertainty regarding the exchange rate. Furthermore, Danish exporters also face uncertainty regarding the exchange rate and could need to engage in costly arrangements with financial intermediaries in order to eliminate this uncertainty when trading with agents abroad. Finally, we ignore issues related to the potential budget discipline being put on the Government in order to keep a peg credible.

Additionally, a number of obvious extensions of this work lies ahead: Firstly, we are currently considering a more generalised form of the Taylor rule, examining the welfare gains attainable when expanding the Taylor rule to include the exchange rate. We need more work on this issue however, since we discovered a large range of spikes and ridges in the welfare levels derived from different parameterisations of the Taylor rule equipped with changes in the exchange rate. At this point we are unable to explain this, but we will seek to get further insight into this area in our future research. Secondly, we should examine the consequences of focusing on conditional moments rather than using unconditional moments. This might be of great importance for the Danish case, since we currently ignore the transition from the peg regime to the Taylor rule regime.

This paper, however, contributes to the ongoing debate in Denmark whether to stick with the peg, and it contributes to the literature in general by performing a welfare analysis on an estimated small open economy DSGE model with a number of nominal and real rigidities.

A The Non-linear Model

$$Q_t^d = \alpha^d \frac{Z_t}{\tilde{P}_t^d}, \quad (15)$$

$$Q_t^m = (1 - \alpha^d) \frac{Z_t}{\tilde{P}_t^m}, \quad (16)$$

$$Q_t^x = (\tilde{P}_t^x)^{-\eta} Y_t^*, \quad (17)$$

$$1 = (\tilde{P}_t^d)^{\alpha^d} (\tilde{P}_t^m)^{1-\alpha^d}, \quad (18)$$

$$L_t = \frac{1-\psi}{\psi} \frac{\tilde{R}_t}{\tilde{W}_t} K_t, \quad (19)$$

$$mc_t = \frac{1}{\theta_t} \tilde{W}_t^{1-\psi} \tilde{R}_t^\psi \psi^{-\psi} (1-\psi)^{-(1-\psi)}, \quad (20)$$

$$\rho_{t,t+1} = \beta (U_{C,t+1}/U_{C,t}) \Pi_{t+1}^{-1}, \quad (21)$$

$$U_{C,t} = \zeta_t^b (C_t - hC_{t-1})^{-\sigma_C}; \quad (22)$$

$$\tilde{\mathcal{N}}_t^d = Q_t^d (1+\nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^{\frac{1+2\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^d \right], \quad (23)$$

$$\tilde{\mathcal{D}}_t^d = Q_t^d + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{D}}_{t+1}^d \right], \quad (24)$$

$$\tilde{P}_t^d = \left[d \left(\tilde{P}_{t-1}^d / \Pi_t \right)^{-\frac{1}{\nu}} + (1-d) \left(\tilde{\mathcal{N}}_t^d / \tilde{\mathcal{D}}_t^d \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (25)$$

$$\tilde{\mathcal{N}}_t^x = Q_t^x (1+\nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x}{\tilde{P}_t^x} \Pi_{t+1}^* \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^x \right], \quad (26)$$

$$\tilde{\mathcal{D}}_t^x = Q_t^x \mathcal{E}_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x}{\tilde{P}_t^x} \right)^{\frac{1+\nu}{\nu}} (\Pi_{t+1}^*)^{\frac{1}{\nu}} \Pi_{t+1} \tilde{\mathcal{D}}_{t+1}^x \right], \quad (27)$$

$$\tilde{P}_t^x = \left[d \left(\tilde{P}_{t-1}^x / \Pi_t^* \right)^{-\frac{1}{\nu}} + (1-d) \left(\tilde{\mathcal{N}}_t^x / \tilde{\mathcal{D}}_t^x \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (28)$$

$$\tilde{\mathcal{N}}_t^m = Q_t^m (1 + \nu) + \frac{d}{1 + i_t^*} E_t \left[\left(\frac{\tilde{P}_{t+1}^m}{\tilde{P}_t^m} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{N}}_{t+1}^m \right] \quad (29)$$

$$\tilde{\mathcal{D}}_t^m = Q_t^m \mathcal{E}_t + \frac{d}{1 + i_t^*} E_t \left[\left(\frac{\tilde{P}_{t+1}^m}{\tilde{P}_t^m} \right)^{\frac{1+\nu}{\nu}} (\Pi_{t+1})^{\frac{1}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{D}}_{t+1}^m \right], \quad (30)$$

$$\tilde{P}_t^m = \left[d \left(\tilde{P}_{t-1}^m / \Pi_t \right)^{-\frac{1}{\nu}} + (1 - d) \left(\tilde{\mathcal{N}}_t^m / \tilde{\mathcal{D}}_t^m \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (31)$$

$$K_{t+1} = K_t (1 - \delta) + I_t - \frac{1}{2} \frac{\Phi (K_{t+1} - K_t)^2}{K_t}, \quad (32)$$

$$\tilde{B}_{t+1} = \left(1 + i_{t-1}^f \right) \tilde{B}_t / \Pi_t^* + \tilde{P}_t^x Q_t^x - \frac{\tilde{P}_t^m}{\mathcal{E}_t} Q_t^m; \quad (33)$$

$$E_t \left[\rho_{t,t+1} \Pi_{t+1} \frac{\left(\tilde{R}_{t+1} + (1 - \delta) - \frac{\Phi}{2} \left(1 - \left(\frac{K_{t+2}}{K_{t+1}} \right)^2 \right) \right)}{\left[1 + \Phi \frac{K_{t+1} - K_t}{K_t} \right]} \right] = 1, \quad (34)$$

$$(1 + i_t) E_t [\rho_{t,t+1}] = 1, \quad (35)$$

$$\left(1 + i_t^f \right) E_t \left[\rho_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \frac{\Pi_{t+1}}{\Pi_{t+1}^*} \right] = 1; \quad (36)$$

$$\tilde{\mathcal{N}}_t^w = \zeta_t^b \zeta_t^L (1 + \gamma) \left(\tilde{W}_t \right)^{\frac{1+\gamma}{\gamma} (1 + \sigma_L)} L_t^{1 + \sigma_L} + d\beta E_t \left[\Pi_{t+1}^{\frac{1+\gamma}{\gamma} (1 + \sigma_L)} \tilde{\mathcal{N}}_{t+1}^w \right], \quad (37)$$

$$\tilde{\mathcal{D}}_t^w = U_{C,t} \tilde{W}_t^{\frac{1+\gamma}{\gamma}} L_t + d\beta E_t \left[\Pi_{t+1}^{\frac{1}{\gamma}} \tilde{\mathcal{D}}_{t+1}^w \right], \quad (38)$$

$$\tilde{w}_{t,t} = \left[\frac{\tilde{\mathcal{N}}_t^w}{\tilde{\mathcal{D}}_t^w} \right]^{\frac{\gamma}{\gamma + (1 + \gamma) \sigma_L}}, \quad (39)$$

$$\tilde{W}_t = \left[D \left(\tilde{W}_{t-1} / \Pi_t \right)^{-\frac{1}{\gamma}} + (1 - D) \tilde{w}_{t,t}^{-\frac{1}{\gamma}} \right]^{-\gamma}; \quad (40)$$

$$\left(1 + i_t^f \right) = \left(1 + i_t^* \right) v_t \exp \left\{ -\lambda \mathcal{E}_t \Pi_{t+1}^* \frac{\tilde{B}_{t+1}}{\Xi} \right\}, \quad \Xi = \mathcal{E} \hat{\mathcal{P}}^x Q^x, \quad (41)$$

$$\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{\Pi_t}{\Pi_t^*} = \frac{\xi_t}{\xi_{t-1}}; \quad (42)$$

$$Y_t = Q_t^d + Q_t^x, \quad (43)$$

$$Z_t = C_t + I_t, \quad (44)$$

$$K_t = \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{\tilde{W}_t}{\tilde{R}_t} \right)^{1-\psi} \left[\left(\frac{\tilde{\mathcal{P}}_t^d}{\tilde{P}_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{\tilde{\mathcal{P}}_t^x}{\tilde{P}_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right], \quad (45)$$

$$\tilde{\mathcal{P}}_t^d = \left[d \left(\tilde{\mathcal{P}}_{t-1}^d / \Pi_t \right)^{-\frac{1+\nu}{\nu}} + (1-d) \left(\tilde{p}_{t,t}^d \right)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad (46)$$

$$\tilde{\mathcal{P}}_t^x = \left[d \left(\tilde{\mathcal{P}}_{t-1}^x / \Pi_t^* \right)^{-\frac{1+\nu}{\nu}} + (1-d) \left(\tilde{p}_{t,t}^x \right)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}; \quad (47)$$

$$\tilde{\mathcal{W}}_t = \left[D \left(\tilde{\mathcal{W}}_{t-1} / \Pi_t \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} + (1-D) \tilde{w}_{t,t}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}, \quad (48)$$

$$SW_t = \frac{\zeta_t^b}{1-\sigma_C} (C_t - h\bar{C}_{t-1})^{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} L_t^{1+\sigma_L} \left(\frac{\tilde{\mathcal{W}}_t}{\tilde{W}_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)}. \quad (49)$$

The processes governing the persistent structural shocks are given as

$$\hat{\zeta}_t^b = \varrho^b \hat{\zeta}_{t-1}^b + \varepsilon_t^b, \quad (50)$$

$$\hat{\zeta}_t^l = \varrho^l \hat{\zeta}_{t-1}^l + \varepsilon_t^l, \quad (51)$$

$$\hat{\theta}_t = \varrho^t \hat{\theta}_{t-1} + \varepsilon_t^t, \quad (52)$$

$$\hat{\xi}_t^j = \varrho_j^m \hat{\xi}_{t-1}^j + \varepsilon_t^m, \quad j = peg, TR \quad (53)$$

$$\hat{i}_t^* = \varrho^i \hat{i}_{t-1}^* + \varepsilon_t^i, \quad (54)$$

$$\hat{P}_t^* = \varrho^P \hat{P}_{t-1}^* + \varepsilon_t^P, \quad (55)$$

$$\hat{Y}_t^* = \varrho^Y \hat{Y}_{t-1}^* + \varepsilon_t^Y, \quad (56)$$

where hats denote relative deviations from the steady state, and $0 < \varrho^j < 1$, cf. Table 1. Since, however, the monetary policy shock under the Taylor regime is assumed to be *i.i.d.*, we have $\varrho_{TR}^m = 0$.

Table 5: Variables and Parameters

<i>Variables</i>		<i>Exogenous Variables</i>	
Z_t	Final goods	θ_t	Technology level in intermediary sector
P_t	Price of Z	ζ_t^b	Preference discount rate shock
Q_t^i	Intermediate goods	ζ_t^l	Labor supply shock
P_t^i	Price of Q^i	v_t	UIP shock
$p_{t,\tau}^i$	Intermediary price optimized in period τ	ξ_t	Exchange-rate policy (peg) shock
\mathcal{P}_t^i	Price dispersion measure	Y_t^*	Foreign GDP
Y_t	GDP ($Q^d + Q^x$)	P_t^*	Foreign price level
R_t	Rental rate of capital	i_t^*	Foreign interest rate
MC_t	Marginal cost in intermediary sector		
e_t	Exchange rate		
$\rho_{t,\tau}$	Discount factor between periods t and τ	<i>Parameters (time invariant)</i>	
$R_{t,\tau}$	Foreign discount factor	ν	Net price markup (intermediaries)
C_t	Final consumption	γ	Net wage markup
L_t	Aggregate labor supply	α^d	Share of Q^d in final output
$w_{t,\tau}$	Wage level optimized in period τ	ψ	Capital share in intermediate goods
W_t	Aggregate wage level	d	Calvo parameter, intermediaries
$l_t(s, j)$	Labor of type j supplied to firm s	β	Utility discount factor
K_t	Capital stock	h	Habit persistence
I_t	Investment	σ_C^{-1}	Household IES
A_t	Domestic bonds (0 in eqm.)	σ_L^{-1}	Work effort elasticity
B_t	Foreign bonds in foreign currency	δ	Capital depreciation rate
i_t	Domestic interest rate	Φ	Capital adjustment cost
i_t^f	Return on B_t to domestic agents	Ξ	ss export in units of Z
Ω_t	Wedge between i_t^* and i_t^f	λ	Debt premium on foreign bonds
χ_t	Compound variable in wage eqn.	D	Calvo parameter, wages
$U_{C,t}$	Marginal utility of consumption	η	Export demand elasticity
$U_{L,t}$	Marginal disutility of labor		
\mathcal{N}_t^i	Auxiliary variable ($p_{t,t}^i$ and $w_{t,t}$)		
\mathcal{D}_t^i	Auxiliary variable ($p_{t,t}^i$ and $w_{t,t}$)		

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